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Mappings that fulfil weak contractions with respect to the F-Invariant Set and the c-distance.

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Keyword

Mixed, Monotone, Cone Metric, Fixed, Theorems.

Abstract

With this research, the author presents a fresh, easy-to-understand, and unified method for studying coupled and tripled fixed point theory. As a broader version of traditional metric spaces, the cone metric space is a relatively new notion. These theorems generalise and extend a number of previously established conclusions with a similar structure in cone metric spaces. The purpose of this study is to extend the findings of linked fixed point theorems to tripled fixed point theorems by providing a definition of F-invariant sets denoted by M when $M \in X^6$. The major conclusion generalise and unify previous findings on triple fixed point theorems, including those of Poom et al.

Introduction

A novel fixed point theorem for a mixed monotone mapping in a metric space was established by Bhaskar and Lakshmikantham. They decided to label it a "coupled fixed point." Recently, Berinde and Borcut [1] have gone even farther by extending this novel idea to threefold fixed point. They proved that contractive mappings in partly ordered full metric spaces exist and are unique. Huang and Zang's 2007 extension of metric space, the cone metric space, is an example of this trend. Substituting a Banach space for the set of real numbers, they expanded the scope of metric spaces. Some scholars have proved a number of triple fixed point theorems in cone metric space in recent years.

By relaxing the idea of the mixed monotone condition, this study generalises the conclusions of Poom et al. linked fixed point theorems to triple fixed point theorems. The major conclusions generalise and bring together several results from the triple fixed point theorems as well as those of Poom et al.

We have following definitions:

Definition 1.1 Let X be a non-empty set. Suppose the mapping $d: X \times X \rightarrow E$ (E is real Banach space) satisfies:

- (i) $0 < d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ iff $x = y$,
- (ii) $d(x, y) = d(y, x)$ for all $x, y \in X$,
- (iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$,

Then d is called a **cone metric** on X and (X, d) is called a **cone metric space**.

Definition 1.2. An element $(x, y) \in X \times X$ is called a **coupled fixed point** of the mapping $\mathcal{F}: X \times X \rightarrow X$, if $x = \mathcal{F}(x, y)$ and $y = \mathcal{F}(y, x)$.

Definition 1.3. Let $\mathcal{F}: X^3 \rightarrow X$. An element (x, y, z) is called a **tripled fixed point** of $\mathcal{F}$ if $\mathcal{F}(x, y, z) = x$, $\mathcal{F}(y, x, z) = y$, $\mathcal{F}(z, y, x) = z$.



Definition 1.4. Let (X, d) be a cone metric space and $\mathcal{F}: X \times X \times X \rightarrow X$ is a given mapping, let M be a non-empty subset of X^4 , one say that M is the $\mathcal{F}$ -invariant subset of X^4 iff, for all $x, y, w \in X$, one has

- (i) $(x, y, z, w) \in M \Leftrightarrow (w, z, y, x) \in M$,
 (ii) $(x, y, z, w) \in M \Leftrightarrow (\mathcal{A}(x, y, z), \mathcal{A}(z, y, x), \mathcal{A}(y, z, w), \mathcal{A}(w, z, y), \mathcal{A}(z, w, x), \mathcal{A}(x, w, z)) \in M$.

Definition 1.5. Let (X, d) be a cone metric space and $\mathcal{F}: X^3 \rightarrow X$ is a given mapping. Let M be a non-empty subset of X^6 . One says that M is the $\mathcal{F}$ -invariant subset of X^6 iff, for all $x, y, z, u, v, w \in X$,

$$\begin{aligned} (x, y, z, u, v, w) \in M &\Leftrightarrow (w, v, u, z, y, x) \in M \\ (x, y, z, u, v, w) \in M &\Rightarrow (\mathcal{A}(x, y, z), \mathcal{A}(y, x, z), \mathcal{A}(z, y, x), \mathcal{A}(u, v, w), \mathcal{A}(v, u, w), \mathcal{A}(w, v, u)) \in M. \end{aligned}$$

Because of its simplicity and utility, Banach's fixed point theorem from 1922 has become a crucial tool in resolving existence issues throughout many subfields of non-linear analysis. Matrix equations were among the areas where Ran and Reurings' [12] generalisation of the Banach contraction principle to partially ordered metric spaces was used. The unique solution to a first order differential equation was obtained by Nieto and L'opez [11], who generalised a result of Ran and Reurings [12] for non-decreasing mappings.

As an extension of metric spaces, the notion of cone metric spaces is introduced, in which every pair of points is associated with a cone-shaped section of a real Banach space. Because of the cone's geometry, Banach spaces have an innate partial order. In their work, Huang and Zhang [5] developed the idea of a cone metric space and created the Banach contraction mapping principle for this domain. Then, several writers have investigated the issue of fixed points in cone metric spaces. Because of the research done on cone-shaped metric spaces.

The concept of a connected fixed point of a mapping F from $X \times X$ into X was first presented by Bhaskar and Lakshmikantham [2]. In order to investigate the existence and uniqueness of a solution for a periodic boundary value issue, they first developed certain connected fixed point findings. developed the idea of linked coincidence points, and demonstrated coupled coincidence and coupled common fixed point findings for mappings F from XX into X , and g from X into X , meeting nonlinear contraction in ordered metric space. In order to go further into linked fixed point theory.

Conclusion:

Banach's fixed point theorem, which was developed in 1922, has evolved into an essential resource for addressing existence problems in a variety of subfields of non-linear analysis due to the fact that it is both straightforward and practical. The generalisation of the Banach contraction principle to partly ordered metric spaces that was done by Ran and Reurings was used in a number of different domains, including matrices equations. The concept of cone metric spaces is proposed as an extension of metric spaces. In these spaces, every pair of points is connected with a cone-shaped portion of a real Banach space. Banach spaces are defined as having an inherent partial order due to the geometry of the cone. Came up with the concept of a cone metric space and devised the Banach contraction mapping principle specifically for this field of study. The problem of fixed points in cone metric spaces has also been researched by a number of authors. because of the studies that were conducted on metric spaces shaped like cones. The first people to describe the idea of a linked fixed point of a mapping F from $X \times X$ into X . They started by developing some related fixed point results before moving on to analyse whether or not a solution to a problem involving periodic boundary values exists and whether or not it is unique. created the concept of connected coincidence points, as well as presented coupled coincidence and coupled common fixed point results for mappings F from XX into X , and g from X into X , while facing nonlinear contraction in ordered metric space. For the purpose of advancing our understanding of linked fixed point theory.

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