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FIBONACCI WAVELET OPERATIONAL MATRIX METHOD FOR A FRACTIONAL DENGUE TRANSMISSION MODEL WITH HUMAN–MOSQUITO DYNAMICS

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Keywords	***Abstract***
Fractional-order epidemic model, Dengue transmission dynamics, Fibonacci wavelets, Operational matrix method, Basic reproduction number, Numerical simulation MSC (2020): 34A08, 92D30, 65M70, 65T60.	This study considers a fractional-order dengue transmission model incorporating hu man and mosquito populations. The human population is classified into susceptible, exposed, infected, and recovered compartments, while mosquitoes are divided into susceptible and infected classes. The model is formulated using Caputo fractional derivatives to account for memory effects inherent in epidemic processes.To compute approximate solutions, a Fibonacci wavelet operational matrix method is employed. The state variables are expanded in terms of Fibonacci wavelets, and the system is reduced to a set of nonlinear algebraic equations, which are solved numerically. Numerical simulations for varying fractional orders are conducted to examine the influence of memory on disease dynamics. The basic reproduction number is also derived to de termine the threshold for disease persistence. The results indicate that the fractional framework enhances modeling flexibility, while the proposed numerical scheme provides an efficient solution.

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Introduction

Mathematical modeling plays a vital role in understanding infectious disease dynamics and designing control strategies by representing transmission processes through differential equations [1, 2]. Dengue fever, a rapidly spreading mosquito-borne disease prevalent in tropical and subtrop ical regions, poses a significant public health challenge [11]. Fractional-order differential equations have gained attention for their ability to incorporate memory effects, providing a more realistic description of epidemic dynamics compared to classical models [4]. Due to the lack of exact solutions, efficient numerical methods are required, and wavelet-based techniques, particularly Fibonacci wavelets, have proven effective in solving fractional-order systems with high accuracy [5]. In this paper, we investigate a fractional SEIR dengue transmission model with incubation delay to better capture the dynamics of infection. Recent studies on fractional-order epidemic models highlight their effectiveness in incorporating memory effects and providing more realistic descriptions of disease spread compared to classical models [16, 8]. Motivated by these develop ments, the Fibonacci wavelet operational matrix method is employed to obtain efficient numerical solutions of the proposed model.Differential equations are fundamental tools for modeling and analyzing a wide range of phenomena in science and engineering. They are commonly used to represent dynamic processes such as population dynamics, heat conduction, fluid motion, and infectious disease spread. In epidemiology, these models have gained significant importance for understanding transmission dynamics, forecasting outbreak patterns, and assessing intervention strategies [18, 1]. Thus, differential equation-based models serve as valuable instruments for informed public health planning and decision-making. Although classical differential equation 1 models have been widely used to describe many physical and biological processes, they sometimes fail to capture memory and hereditary effects present in real-world systems. To overcome this limitation, fractional differential equations have been introduced as a powerful generalization of classical models. Fractional-order models have been successfully applied in various fields such as viscoelasticity, control theory, signal processing, and epidemiology, where they provide a more accurate description of complex dynamical behaviors [9, 4]. Consequently, fractional epidemic models have attracted considerable attention for studying the spread of infectious diseases. In recent years, data-driven approaches have become increasingly important in epidemic modeling, where real epidemiological data such as infection, recovery, and mortality rates are used to estimate model parameters and improve prediction accuracy. These models allow researchers to analyze the spread of infectious diseases and evaluate possible control strategies during epidemic outbreaks. By combining mathematical modeling with real-world data, epidemic models provide valuable insights into disease transmission dynamics and help guide public health decision-making [18, 1]. The proposed fractional SEIR dengue transmission model is inherently nonlinear due to the interaction terms representing disease transmission between susceptible and infected individuals. Analytical solutions of such nonlinear fractional systems are generally difficult to obtain; therefore, efficient numerical techniques are required. In this study, the Fibonacci wavelet operational matrix approach is utilized to compute approximate solutions of the proposed model. This technique converts the fractional differential system into a set of algebraic equations, which can be solved efficiently. Such mathematical models are useful for understanding disease transmission dynamics and can assist researchers and public health authorities in predicting possible future outbreaks and designing effective control strategies [18, 1]. Dengue fever is a mosquito-borne viral infection and a major public health concern in tropical and subtropical regions, primarily transmitted by Aedes aegypti mosquitoes and caused by four related serotypes [11, 12]. Its rapid spread, driven

by urbanization, globalization, and climate change, and its wide clinical spectrum, from mild fever to severe complications, highlight its epidemiological importance. Various mathematical approaches have been used to study dengue dynamics: Yoda et al. analyzed an SEIR–SI model with stability and reproduction number considerations [13], Meetei et al. employed a fractional model with Caputo–Fabrizio derivatives [8], and Rahman et al. developed a fractional piecewise model with singular and non-singular kernels [14]. Motivated by these studies, we consider a fractional SEIR dengue model with incubation delay and apply the Fibonacci wavelet operational matrix method to obtain efficient numerical solutions. Mathematical modeling has also been widely applied to study the transmission dynamics of several infectious diseases such as COVID–19, Ebola, and malaria. For instance, Area et al. developed a fractional-order epidemic model to investigate the spread of Ebola and analyzed the qualitative behavior of the system using numerical simulations [15]. In a similar study, Khan et al. investigated a fractional SEIR model for infectious diseases, analyzing its stability charac teristics and dynamic behavior using numerical techniques [16]. These studies demonstrate the effectiveness of fractional differential equation models in describing epidemic processes. Motivated by these contributions, the present work focuses on a fractional SEIR dengue transmission model with incubation delay and employs the Fibonacci wavelet operational matrix method to obtain approximate numerical solutions. In recent years, infectious diseases such as COVID–19, Ebola, Zika, and dengue have emerged as major global health threats due to their rapid transmission and significant public health impact, prompting extensive research in epidemiology and mathematical modeling [19, 18]. Mathemat ical models play a crucial role in analyzing disease dynamics and evaluating control strategies. Researchers across disciplines, including epidemiology, mathematics, and public health, have de veloped computational models to estimate key epidemiological parameters and assess interventions such as vaccination, quarantine, and vector control. These models support policymakers in design-ing effective responses and predicting future outbreaks, thereby contributing to improved disease prevention and management (18, 1].

The objective of this study is to analyze dengue transmission dynamics using a fractional SEIR model with incubation delay, incorporating memory effects for a more realistic representation. Due to the analytical intractability of the nonlinear system, the Fibonacci wavelet operational matrix method is employed to obtain approximate solutions by transforming the model into a system of algebraic equations. The results provide useful insights into dengue dynamics and support effective prediction and management of outbreaks.

The structure of the paper is outlined as follows. Section 2 introduces essential concepts and definitions from fractional calculus and Fibonacci wavelets. In Section 3, the fractional. SEIR, dengue model with incubation delay is developed. Section 4 presents the Fibonacci wavelet operational matrix technique for computing numerical solutions. Section 5 provides simulation results along with discussions to demonstrate the performance of the method. Finally, Section 6 concludes the study and highlights potential directions for future research.

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2. Fractional-order SEIR Dengue Transmission Model with Mosquito Dynamics

To develop the fractional-order model, the Riemann–Liouville fractional integral and the Caputo fractional derivative are utilized as introduced earlier. For $\alpha > 0$, the Riemann–Liouville fractional integral of a function $g(x) \in L^1[a,b]$ is defined as [9]:

$$J_a^\alpha g(x) = \begin{cases} g(x), & \alpha = 0, \\ \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_a^x (x-t)^{\alpha-1} g(t)\, dt, & \alpha > 0,\ x > a, \end{cases} \tag{2.1}$$

Similarly, the Caputo fractional derivative for $g(x) \in C^n$ is given by [?]:

$$D_a^\alpha g(x) = \begin{cases} g^{(n)}(x), & \alpha = n \in \mathbb{N}, \\ \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_a^x \frac{g^{(n)}(t)}{(x-t)^{\alpha+1-n}}\, dt, & n-1 < \alpha < n,\ n \in \mathbb{N}. \end{cases} \tag{2.2}$$

Accordingly, for $0 < \alpha < 1$, the fractional SEIR dengue transmission model with mosquito dynamics is expressed by the following system:

$$\left.\begin{aligned} \frac{d^\alpha S_h}{dt^\alpha} &= \Lambda_h - \beta_h S_h I_m - \mu_h S_h, \\ \frac{d^\alpha E_h}{dt^\alpha} &= \beta_h S_h I_m - (\sigma_h + \mu_h) E_h, \\ \frac{d^\alpha I_h}{dt^\alpha} &= \sigma_h E_h - (\gamma_h + \mu_h) I_h, \\ \frac{d^\alpha R_h}{dt^\alpha} &= \gamma_h I_h - \mu_h R_h, \\ \frac{d^\alpha S_m}{dt^\alpha} &= \Lambda_m - \beta_m S_m I_h - \mu_m S_m, \\ \frac{d^\alpha I_m}{dt^\alpha} &= \beta_m S_m I_h - \mu_m I_m \end{aligned}\right\}. \tag{2.3}$$

Assuming that the total human and mosquito populations remain constant, the initial populations satisfy

$$S_h(0) + E_h(0) + I_h(0) + R_h(0) = N_h,$$

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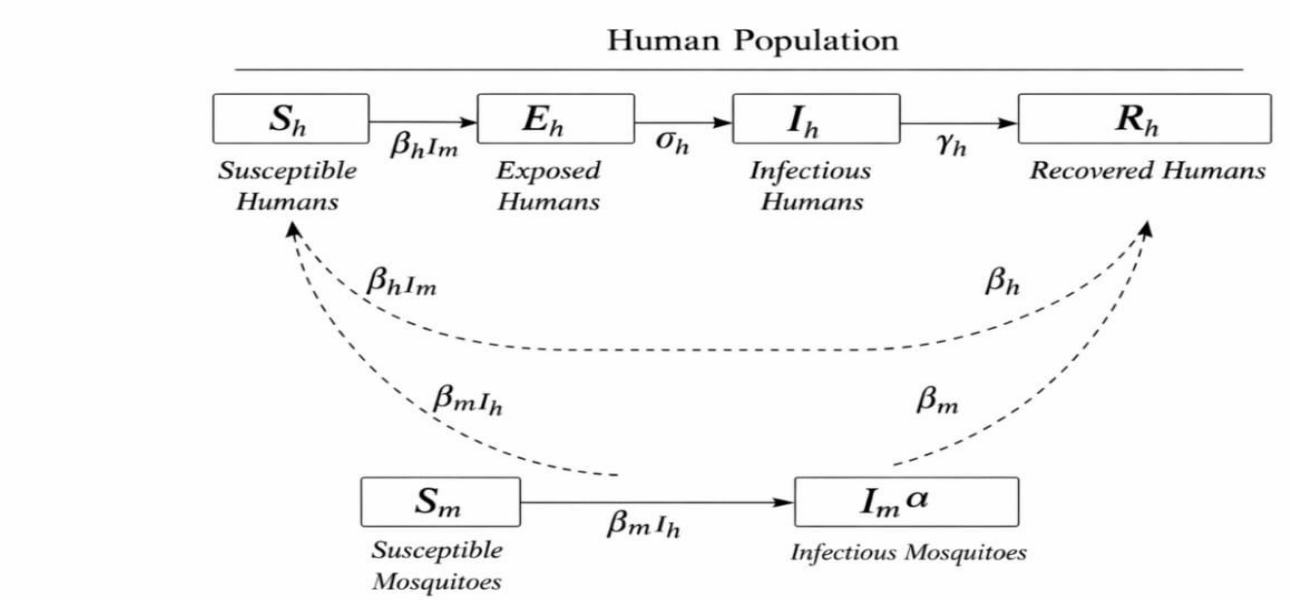

FIGURE 1. Schematic diagram of the fractional-order dengue transmission model illustrating the interaction between human and mosquito populations.

$$S_m(0) + I_m(0) = N_m.$$

All state variables are assumed to be nonnegative for $t \geq 0$.

3. Fibonacci Wavelet Operational Matrix Method

In this section, the Fibonacci Wavelet Operational Matrix Method (FWOMM) is developed to compute numerical solutions of the nonlinear fractional-order dengue model presented in Section 3. Due to their orthogonality, compact support, and rapid convergence, wavelet-based operational matrix methods are well suited for fractional differential equations. In particular, Fibonacci wavelets provide an efficient basis for approximating nonlinear systems with memory effects, making them suitable for fractional epidemic models where past states influence current dynamics.

3.1. **Wavelets.** Wavelets are functions generated by scaling and translation of a prototype known as the *mother wavelet*. Their localization in both time and frequency domains makes them effective for analyzing nonlocal and memory-dependent phenomena. As a result, wavelet-based methods are widely used for solving fractional differential equations with high accuracy and efficiency [21, 22]. A function $\psi \in L^2(\mathbb{R})$ is called an admissible wavelet if it satisfies the admissibility

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Parameter	Name	Units	Value	Description
$S_h(0)$	Initial susceptible humans	Individuals	150	Initial number of humans susceptible to dengue infection
$E_h(0)$	Initial exposed humans	Individuals	40	Humans infected but not yet infectious at the initial time
$I_h(0)$	Initial infected humans	Individuals	30	Humans capable of transmitting dengue to mosquitoes at the initial time
$R_h(0)$	Initial recovered humans	Individuals	10	Humans recovered from dengue infection with temporary immunity
$S_m(0)$	Initial susceptible mosquitoes	Mosquitoes	200	Mosquitoes capable of becoming infected
$I_m(0)$	Initial infected mosquitoes	Mosquitoes	20	Mosquitoes capable of transmitting dengue to humans
Λ_h	Human recruitment rate	day^{-1}	0.145	Rate at which new humans enter the susceptible population
Λ_m	Mosquito recruitment rate	day^{-1}	0.25	Birth rate of mosquitoes entering the susceptible class
β_h	Mosquito $\rightarrow$ human transmission	day^{-1}	0.00038	Transmission rate at which humans become infected after mosquito bites
β_m	Human $\rightarrow$ mosquito transmission	day^{-1}	0.0021	Transmission rate at which mosquitoes become infected after biting infected humans
μ_h	Human natural death rate	day^{-1}	0.00041	Natural mortality rate of humans
μ_m	Mosquito natural death rate	day^{-1}	0.02	Natural mortality rate of mosquitoes
σ_h	Incubation rate	day^{-1}	0.00211	Rate at which exposed humans become infectious
γ_h	Recovery rate	day^{-1}	0.0169	Rate at which infected humans recover from dengue

TABLE 1. Description and parameter values used in the fractional-order SEIR dengue transmission model.

condition:

$$C_\psi = 2\pi \int_{\mathbb{R}} \frac{|\mathcal{F}\psi(\omega)|^2}{|\omega|}\, d\omega < \infty, \tag{3.1}$$

where $\mathcal{F}\psi$ denotes the Fourier transform of ψ. This admissibility condition ensures that the wavelet transform is invertible.
An important consequence of this condition is that the wavelet has zero mean, that is

$$\int_{\mathbb{R}} \psi(t)\, dt = 0.$$

By applying dilation and translation to the mother wavelet $\psi(t)$, a set of functions known as *daughter wavelets* is generated and defined as [23].

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$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right), \quad a > 0,\ b \in \mathbb{R}. \tag{3.2}$$

Here the parameter a controls the scale of the wavelet, while b determines its translation in time.

3.2. **Fibonacci Wavelets and Function Approximation.** For $x \in \mathbb{R}^+$, the Fibonacci polynomials $\widetilde{P}_m(x)$ are defined recursively by

$$\widetilde{P}_{m+2}(x) = x\widetilde{P}_{m+1}(x) + \widetilde{P}_m(x), \quad m \geq 0, \tag{3.3}$$

with initial conditions

$$\widetilde{P}_0(x) = 0, \qquad \widetilde{P}_1(x) = 1.$$

Based on these polynomials, Fibonacci wavelets over $[0,1]$ are constructed as

$$\Psi_{n,m}(x) = \begin{cases} 2^{\frac{k-1}{2}}\dfrac{1}{\sqrt{w_m}}\widetilde{P}_m\left(2^{k-1}x - n + 1\right), & \dfrac{n-1}{2^{k-1}} \leq x < \dfrac{n}{2^{k-1}}, \\ 0, & \text{otherwise}, \end{cases} \tag{3.4}$$

where k denotes the resolution level and n represents the translation parameter.
Any function $g(x) \in L^2[0,1]$ can be approximated using Fibonacci wavelets as

$$g(x) \approx \sum_{n=1}^{2^k}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x) = \mathbf{C}^T\mathbf{\Psi}(x), \tag{3.5}$$

where $\mathbf{C}$ is the coefficient vector and $\mathbf{\Psi}(x)$ is the basis vector.

3.3. **Operational Matrix of Fractional Integration.** To derive the operational matrix of fractional integration, block pulse functions on $[0,1]$ are introduced as

$$b_\ell(x) = \begin{cases} 1, & \ell r \leq x < (\ell+1)r, \\ 0, & \text{otherwise}, \end{cases} \tag{3.6}$$

where $r = 1/N$ and $\ell = 0, 1, \ldots, N-1$.
Any $f(x) \in L^2[0,1]$ can be approximated in terms of block pulse functions by

$$f(x) \approx \sum_{\ell=0}^{N-1} a_\ell b_\ell(x) = \mathbf{A}^T\mathbf{B}_N(x), \tag{3.7}$$

where $\mathbf{A}$ is the coefficient vector and $\mathbf{B}_N(x)$ is the block pulse basis.
The fractional integration operator acting on $\mathbf{B}_N(x)$ is approximated by

$$(I^\alpha\mathbf{B}_N)(x) \approx \mathbf{J}^\alpha\mathbf{B}_N(x), \tag{3.8}$$

where $\mathbf{J}^\alpha$ denotes the corresponding operational matrix.
Using the relation between Fibonacci wavelets and block pulse functions, the operational matrix for Fibonacci wavelets is given by

$$\mathbf{P}^\alpha_{n,m} = \mathbf{\Psi}_{n,m}\,\mathbf{J}^\alpha\,[\mathbf{\Psi}_{n,m}]^{-1}. \tag{3.9}$$

This matrix formulation allows the fractional differential system of the dengue model to be converted into a system of algebraic equations, which can be solved numerically.

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3.4. **Fibonacci Wavelet Basis and Basis Matrices.** To implement the Fibonacci wavelet operational matrix method, the unknown state variables are expanded in terms of Fibonacci wavelet basis functions. Let k be the resolution level and M the number of modes, with basis vector

$$\mathbf{\Psi}(x) = [\psi_{1,0}(x), \ldots, \psi_{1,M-1}(x), \ldots, \psi_{2^{k-1},0}(x), \ldots, \psi_{2^{k-1},M-1}(x)]^T.$$

For numerical computation, the basis is evaluated at collocation points $x_\ell = \frac{2\ell-1}{2^k M}$, $\ell = 1, 2, \ldots, N$, where $N = 2^{k-1}M$. The corresponding basis matrix is

$$\Psi_{N\times N} = \left[\psi_j(x_i)\right]_{i,j=1}^{N},$$

which, for example, reduces to an 8×8 matrix when $k = 2$ and $M = 4$. In general, this matrix of dimension $N = 2^{k-1}M$ is fundamental in constructing operational matrices of fractional integration, enabling the transformation of the fractional system into a system of algebraic equations.

3.5. **Disease-Free Equilibrium and Basic Reproduction Number.** In epidemiological studies, the basic reproduction number R_0 is a key threshold parameter used to assess whether an infection can invade and persist in a population. It quantifies the expected number of new cases produced by a single infectious individual in a wholly susceptible population.

For the dengue transmission model under consideration, the disease-free equilibrium (DFE) corresponds to the absence of infection in both human and mosquito populations. This is obtained by setting

$$E_h = 0, \quad I_h = 0, \quad R_h = 0, \quad I_m = 0.$$

Hence, the DFE of system (2.3) is given by

$$E_0 = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, 0, \frac{\Lambda_m}{\mu_m}, 0\right).$$

Applying the next-generation matrix method, the basic reproduction number is derived as

$$R_0 = \sqrt{\frac{\beta_h \beta_m \Lambda_h \Lambda_m}{\mu_h \mu_m (\gamma_h + \mu_h)}}.$$

The magnitude of R_0 determines the disease outcome: if $R_0 < 1$, the infection eventually disappears and the DFE remains stable, whereas if $R_0 > 1$, the disease persists and may become endemic.

4. Solution of the Fractional-order SEIR Dengue Transmission Model

To solve the fractional-order dengue transmission model, the Fibonacci wavelet method is employed to transform the system of fractional differential equations into a set of nonlinear algebraic equations through wavelet basis expansion. The unknown coefficients are obtained by solving this algebraic system, yielding an efficient and accurate numerical scheme that effectively captures memory effects inherent in fractional models. Accordingly, we consider the fractional-order dengue transmission model with mosquito dynamics given by:

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$$\left.\begin{aligned}
\frac{d^\alpha S_h}{dt^\alpha} &= \Lambda_h - \beta_h S_h I_m - \mu_h S_h,\\
\frac{d^\alpha E_h}{dt^\alpha} &= \beta_h S_h I_m - (\sigma_h + \mu_h) E_h,\\
\frac{d^\alpha I_h}{dt^\alpha} &= \sigma_h E_h - (\gamma_h + \mu_h) I_h,\\
\frac{d^\alpha R_h}{dt^\alpha} &= \gamma_h I_h - \mu_h R_h,\\
\frac{d^\alpha S_m}{dt^\alpha} &= \Lambda_m - \beta_m S_m I_h - \mu_m S_m,\\
\frac{d^\alpha I_m}{dt^\alpha} &= \beta_m S_m I_h - \mu_m I_m.
\end{aligned}\right\} \tag{4.1}$$

subject to the initial conditions

$$\big(S_h(0), E_h(0), I_h(0), R_h(0), S_m(0), I_m(0)\big) = \big(S_{h0}, E_{h0}, I_{h0}, R_{h0}, S_{m0}, I_{m0}\big). \tag{4.2}$$

The highest-order fractional derivatives appearing in system (4.1) are approximated using the Fibonacci wavelet basis as follows:

$$\left.\begin{aligned}
\frac{d^\alpha S_h}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} a_\ell \, \Phi_\ell(t),\\
\frac{d^\alpha E_h}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} b_\ell \, \Phi_\ell(t),\\
\frac{d^\alpha I_h}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} c_\ell \, \Phi_\ell(t),\\
\frac{d^\alpha R_h}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} d_\ell \, \Phi_\ell(t),\\
\frac{d^\alpha S_m}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} e_\ell \, \Phi_\ell(t),\\
\frac{d^\alpha I_m}{dt^\alpha} &= \sum_{\ell=1}^{2^{k-1}M} f_\ell \, \Phi_\ell(t).
\end{aligned}\right\} \tag{4.3}$$

where $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell$, $\ell = 1, 2, \ldots, 2^{k-1}M$, denote the unknown Fibonacci wavelet coefficients. Applying the fractional integral operator to both sides of (4.3) and using the properties of the Caputo fractional derivative, we obtain

$$\begin{aligned}
S_h(t) &= S_{h0} + J_t^\alpha \, {}_0^C D_t^\alpha S_h(t),\\
E_h(t) &= E_{h0} + J_t^\alpha \, {}_0^C D_t^\alpha E_h(t),\\
I_h(t) &= I_{h0} + J_t^\alpha \, {}_0^C D_t^\alpha I_h(t),\\
R_h(t) &= R_{h0} + J_t^\alpha \, {}_0^C D_t^\alpha R_h(t),\\
S_m(t) &= S_{m0} + J_t^\alpha \, {}_0^C D_t^\alpha S_m(t),\\
I_m(t) &= I_{m0} + J_t^\alpha \, {}_0^C D_t^\alpha I_m(t).
\end{aligned}$$

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Using the operational matrix of fractional integration corresponding to Fibonacci wavelets, we obtain

$$\left.\begin{aligned}
S_h(t) &= S_{h0} + \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t),\\
E_h(t) &= E_{h0} + \sum_{\ell=1}^{2^{k-1}M} b_\ell P_\ell^\alpha(t),\\
I_h(t) &= I_{h0} + \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t),\\
R_h(t) &= R_{h0} + \sum_{\ell=1}^{2^{k-1}M} d_\ell P_\ell^\alpha(t),\\
S_m(t) &= S_{m0} + \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t),\\
I_m(t) &= I_{m0} + \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t).
\end{aligned}\right\} \tag{4.4}$$

Substituting (4.3) and (4.4) into the system (4.1), we obtain the following system of nonlinear algebraic equations:

$$\left.\begin{aligned}
\sum a_\ell \Phi_\ell(t) &= \Lambda_h - \mu_h\Big(S_{h0} + \sum a_\ell P_\ell^\alpha(t)\Big) - \beta_h\Big(S_{h0} + \sum a_\ell P_\ell^\alpha(t)\Big)\Big(I_{m0} + \sum f_\ell P_\ell^\alpha(t)\Big),\\
\sum b_\ell \Phi_\ell(t) &= \beta_h\Big(S_{h0} + \sum a_\ell P_\ell^\alpha(t)\Big)\Big(I_{m0} + \sum f_\ell P_\ell^\alpha(t)\Big) - (\sigma_h + \mu_h)\Big(E_{h0} + \sum b_\ell P_\ell^\alpha(t)\Big),\\
\sum c_\ell \Phi_\ell(t) &= \sigma_h\Big(E_{h0} + \sum b_\ell P_\ell^\alpha(t)\Big) - (\gamma_h + \mu_h)\Big(I_{h0} + \sum c_\ell P_\ell^\alpha(t)\Big),\\
\sum d_\ell \Phi_\ell(t) &= \gamma_h\Big(I_{h0} + \sum c_\ell P_\ell^\alpha(t)\Big) - \mu_h\Big(R_{h0} + \sum d_\ell P_\ell^\alpha(t)\Big),\\
\sum e_\ell \Phi_\ell(t) &= \Lambda_m - \mu_m\Big(S_{m0} + \sum e_\ell P_\ell^\alpha(t)\Big) - \beta_m\Big(S_{m0} + \sum e_\ell P_\ell^\alpha(t)\Big)\Big(I_{h0} + \sum c_\ell P_\ell^\alpha(t)\Big),\\
\sum f_\ell \Phi_\ell(t) &= \beta_m\Big(S_{m0} + \sum e_\ell P_\ell^\alpha(t)\Big)\Big(I_{h0} + \sum c_\ell P_\ell^\alpha(t)\Big) - \mu_m\Big(I_{m0} + \sum f_\ell P_\ell^\alpha(t)\Big).
\end{aligned}\right\} \tag{4.5}$$

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By further simplifying (4.5) , we have

$$\left.\begin{aligned}
&\sum_{\ell=1}^{2^{k-1}M} a_\ell \Phi_\ell(t) + \beta_h \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t) \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t) + \beta_h I_{m0} \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t) \\
&\quad + \beta_h S_{h0} \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t) + \mu_h \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t) = \Lambda_h - \beta_h S_{h0} I_{m0} - \mu_h S_{h0}, \\
&\sum_{\ell=1}^{2^{k-1}M} b_\ell \Phi_\ell(t) - \beta_h \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t) \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t) - (\beta_h S_{h0}) \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t) \\
&\quad - (\beta_h I_{m0}) \sum_{\ell=1}^{2^{k-1}M} a_\ell P_\ell^\alpha(t) + (\sigma_h + \mu_h) \sum_{\ell=1}^{2^{k-1}M} b_\ell P_\ell^\alpha(t) = \beta_h S_{h0} I_{m0} - (\sigma_h + \mu_h) E_{h0}, \\
&\sum_{\ell=1}^{2^{k-1}M} c_\ell \Phi_\ell(t) - \sigma_h \sum_{\ell=1}^{2^{k-1}M} b_\ell P_\ell^\alpha(t) + (\gamma_h + \mu_h) \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) = \sigma_h E_{h0} - (\gamma_h + \mu_h) I_{h0}, \\
&\sum_{\ell=1}^{2^{k-1}M} d_\ell \Phi_\ell(t) - \gamma_h \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) + \mu_h \sum_{\ell=1}^{2^{k-1}M} d_\ell P_\ell^\alpha(t) = \gamma_h I_{h0} - \mu_h R_{h0}, \\
&\sum_{\ell=1}^{2^{k-1}M} e_\ell \Phi_\ell(t) + \beta_m \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t) \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) + \beta_m I_{h0} \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t) \\
&\quad + \beta_m S_{m0} \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) + \mu_m \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t) = \Lambda_m - \beta_m S_{m0} I_{h0} - \mu_m S_{m0}, \\
&\sum_{\ell=1}^{2^{k-1}M} f_\ell \Phi_\ell(t) - \beta_m \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t) \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) - (\beta_m S_{m0}) \sum_{\ell=1}^{2^{k-1}M} c_\ell P_\ell^\alpha(t) \\
&\quad - (\beta_m I_{h0}) \sum_{\ell=1}^{2^{k-1}M} e_\ell P_\ell^\alpha(t) + \mu_m \sum_{\ell=1}^{2^{k-1}M} f_\ell P_\ell^\alpha(t) = \beta_m S_{m0} I_{h0} - \mu_m I_{m0}.
\end{aligned}\right\} \tag{4.6}$$

By employing the collocation points, the model is reduced to a system of nonlinear algebraic equations in terms of the Fibonacci wavelet coefficients $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell$, and f_ℓ, where $\ell = 1, 2, \ldots, 2^{k-1}M$. This system is solved numerically using Newton's iterative method implemented in MATLAB. Once the coefficients are obtained, substituting them into (4.4) provides approximate solutions for the state variables, namely the susceptible, exposed, infected, and recovered human populations $S_h(t), E_h(t), I_h(t), R_h(t)$, along with the susceptible and infected mosquito populations $S_m(t)$ and $I_m(t)$.

5. Numerical Results and Discussion

To examine the dynamics of the proposed fractional-order dengue model, a Fibonacci wavelet-based numerical scheme is applied to simulate the temporal evolution of human and mosquito populations, as illustrated in Figures 2–14. The human population is represented by $S_h(t)$, $E_h(t)$, $I_h(t)$, and $R_h(t)$, while the mosquito population is described by $S_m(t)$ and $I_m(t)$. Simulations are carried out for fractional orders $\alpha = 0.4, 0.6, 0.8$, and 1 to evaluate the influence of fractional dynamics on dengue transmission.

Time-dependent evolution plots, sensitivity analyses of infected populations with respect to the fractional order, and three-dimensional surface visualizations in terms of time and fractional order are presented. The parameter values employed in the simulations are listed in Table 1, and the initial conditions are specified accordingly.

These numerical experiments provide a comprehensive framework for investigating the dynamical behavior of the fractional dengue model and for assessing the impact of the fractional parameter on disease propagation.

The dynamical behavior of the proposed fractional dengue transmission model is illustrated in Figures 2–5, where the trajectories of the human and mosquito populations are plotted for different values of the fractional order α. These figures demonstrate how variations in the fractional parameter influence the temporal evolution of the system.

For $\alpha = 0.4$, as shown in Figure 2, the system evolves gradually and smoothly, indicating that the transmission of dengue progresses slowly due to stronger memory effects associated with fractional dynamics. The changes in both human and mosquito populations occur progressively over time.

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When $\alpha = 0.6$, corresponding to Figure 3, the evolution of the system becomes more noticeable and the transitions between population classes occur at a faster rate compared to the previous case.

For $\alpha = 0.8$, illustrated in Figure 4, the trajectories exhibit sharper variations and the infection spreads more rapidly among the populations.

Finally, Figure 5 represents the case $\alpha = 1$, which corresponds to the classical integer-order model. In this case, the system responds most rapidly, showing the fastest progression of the epidemic dynamics. Overall, these figures indicate that smaller values of the fractional order lead to slower and smoother evolution of the system, while larger values of α result in faster transmission dynamics.

The influence of the fractional-order parameter α on the individual population classes is further illustrated through the three-dimensional surface plots presented in Figures 6–11. These plots provide a comprehensive visualization of how each population compartment varies simultaneously with respect to time t and the fractional order α.

Figure 6 depicts the surface behavior of the susceptible human population $S_h(t)$. It can be observed that for smaller values of α, the decline in the susceptible population occurs gradually over time, whereas larger values of α lead to a faster reduction in susceptible individuals due to increased infection transmission.

The exposed human population $E_h(t)$ is shown in Figure 7. For lower fractional orders, the exposed class grows slowly and reaches its peak at later times. As the fractional order increases, the exposed population rises more rapidly, indicating quicker progression from the susceptible to exposed class.

Figure 8 presents the surface plot of the infected human population $I_h(t)$. The infection level changes smoothly for smaller values of α, while larger fractional orders produce steeper surfaces and faster epidemic growth.

The recovered human population $R_h(t)$ is illustrated in Figure 9. The results indicate that the recovered population increases gradually when α is small, whereas higher values of α accelerate the recovery dynamics.

Figures 10 and 11 display the surface plots of the mosquito populations. Figure 10 represents the susceptible mosquito population $S_m(t)$, while Figure 11 illustrates the infected mosquito population $I_m(t)$. Similar trends are observed, where smaller fractional orders result in smoother population variations, whereas larger values of α lead to faster mosquito infection dynamics.

The effect of the fractional-order parameter on the infected populations is further illustrated in Figures 12 and 13. Figure 12 shows the influence of α on the infected human population $I_h(t)$, while Figure 13 presents the corresponding behavior for the infected mosquito population $I_m(t)$. It can be observed that increasing the fractional order accelerates the spread of infection, resulting in sharper and earlier peaks in the infected populations. Conversely, smaller fractional orders lead to slower and smoother variations in the infection levels. These observations highlight the significant role of the fractional parameter in controlling the rate of epidemic propagation.

The variation of the basic reproduction number R_0 with respect to the transmission rate β is shown in Fig. 14. It is observed that R_0 increases with β, indicating that higher transmission enhances disease spread. When $R_0 < 1$, the disease dies out, whereas for $R_0 > 1$, it persists and may lead to an outbreak.

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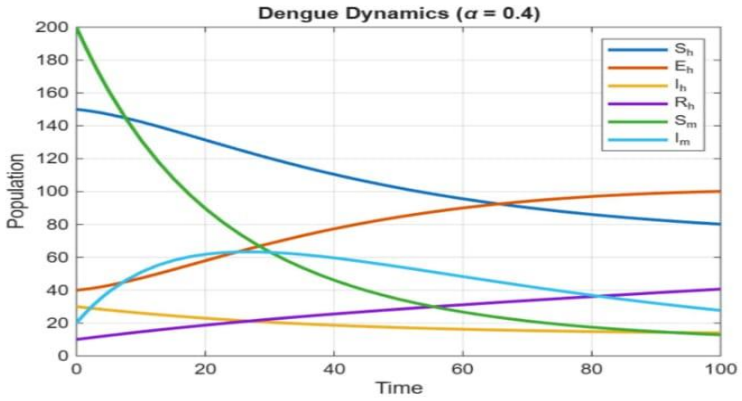

FIGURE 2. Behavior of the model variables at time t using Fibonacci wavelets with $\alpha = 0.4$.

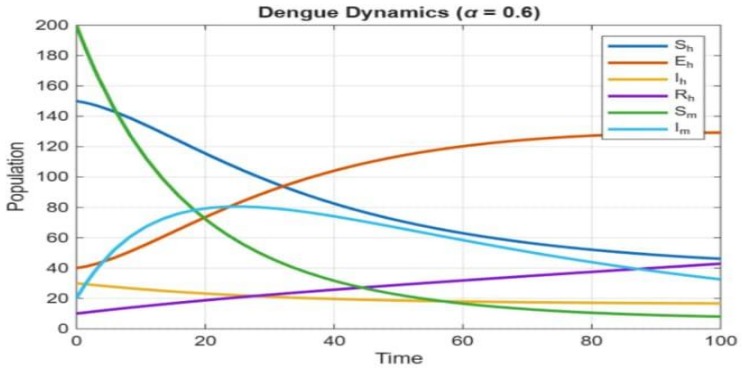

FIGURE 3. Behaviour oif the model variables at time t usin Fivonacci wavelet with $\alpha = 0.6$

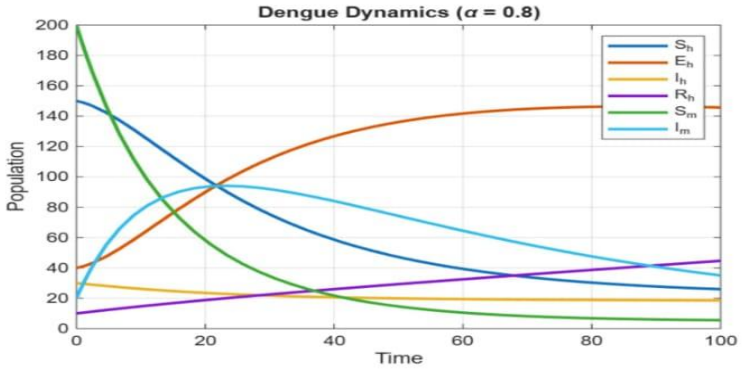

FIGURE 4. Behaviour oif the model variables at time t usin Fivonacci wavelet with $\alpha = 0.8$

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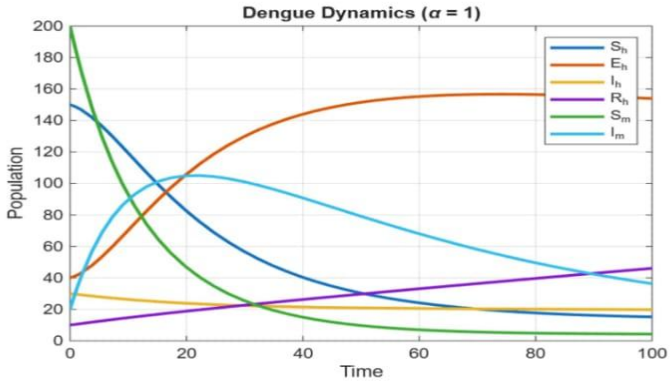

FIGURE 5. Behaviour oif the model variables at time t usin Fivonacci wavelet with $\alpha = 1$

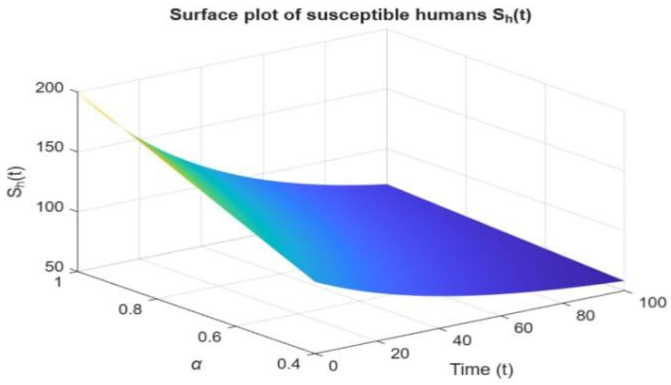

FIGURE 6. Three-dimensional surface of the susceptible human population $S_h(t)$ versus the fractional order α and time t.

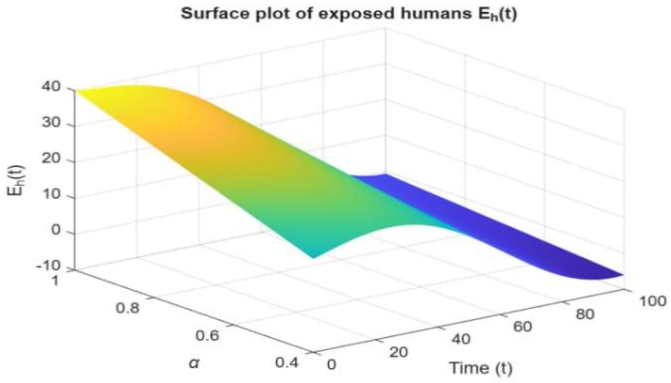

FIGURE 7. Three-dimensional surface of the exposed human population $E_h(t)$ versus the fractional order α and time t.

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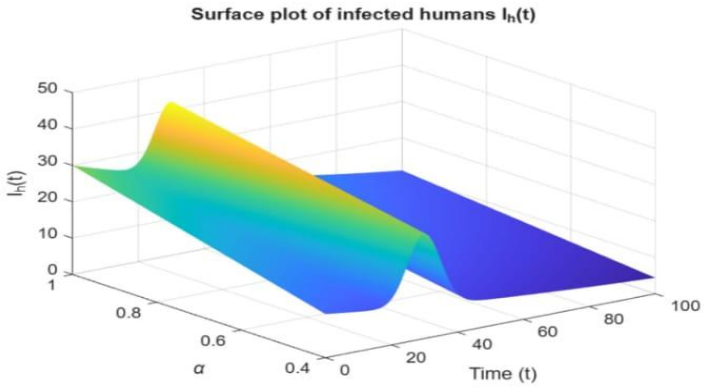

FIGURE 8. Three-dimensional surface of the infected human population $I_h(t)$ versus the fractional order α and time t.

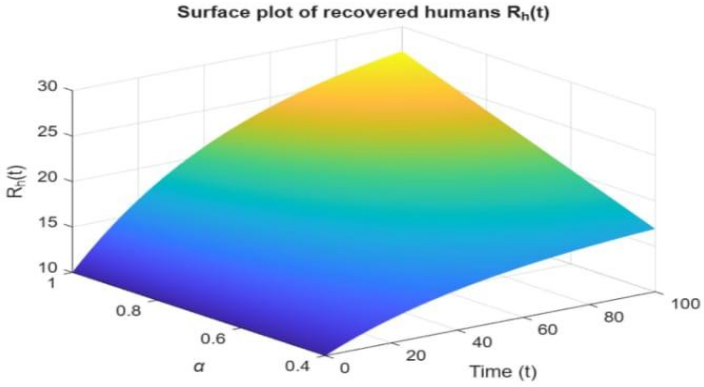

FIGURE 9. Three-dimensional surface of the recovered human population $R_h(t)$ versus the fractional order α and time t.

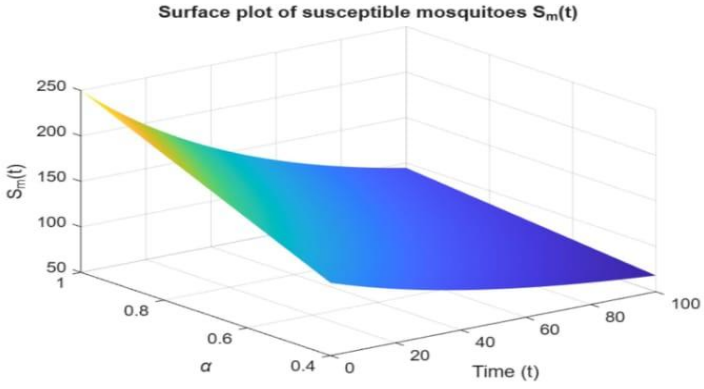

FIGURE 10. Three-dimensional surface of the susceptible mosquito population $S_m(t)$ versus the fractional order α and time t.

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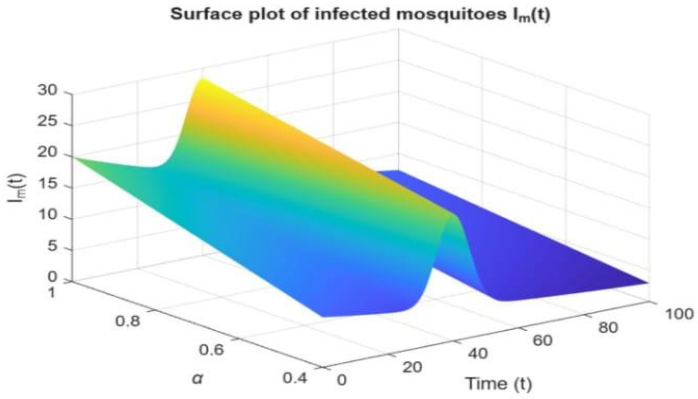

FIGURE 11. Three-dimensional surface of the infected mosquito population $I_m(t)$ versus the fractional order α and time t.

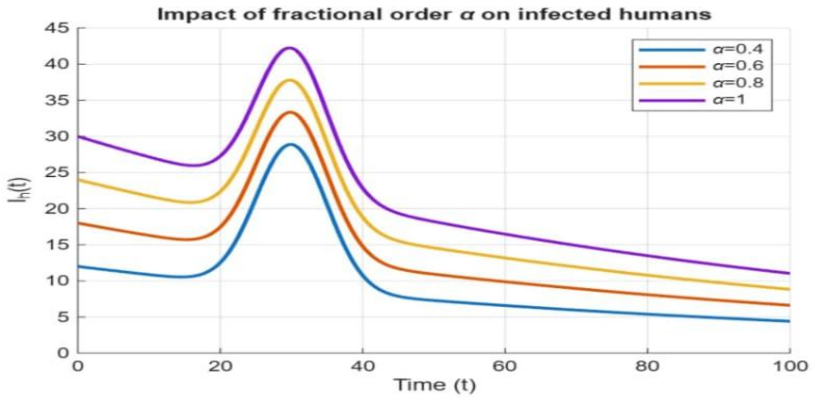

FIGURE 12. Impact of the fractional-order parameter α on the infected human population $I_h(t)$.

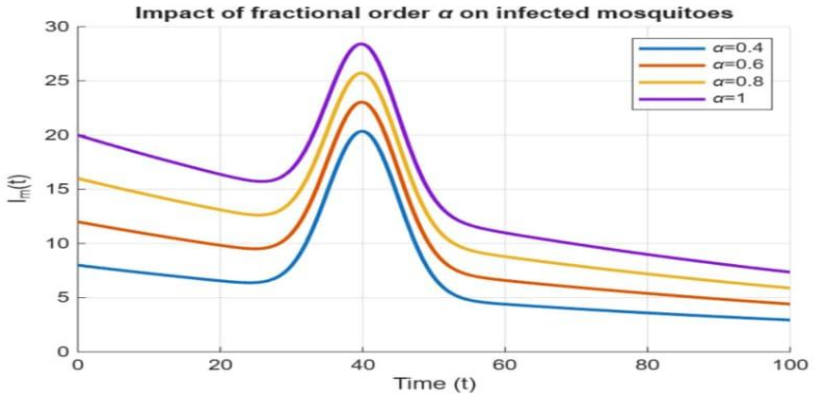

FIGURE 13. Impact of the fractional-order parameter α on the infected mosquito population $I_m(t)$.

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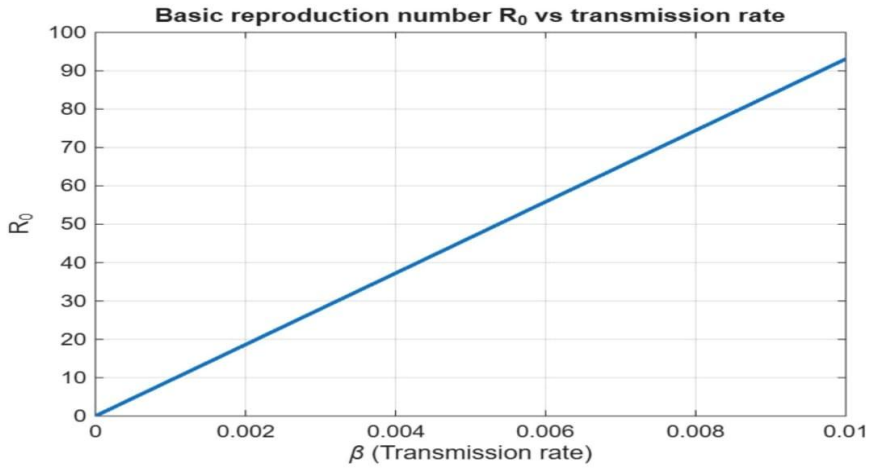

FIGURE 14. Variation of the basic reproduction number R_0 with respect to the transmission rate β.

6. CONCLUSION

This study develops a fractional-order dengue transmission model involving human and mosquito populations, formulated with six compartments and Caputo fractional derivatives to capture memory effects. The nonlinear system is solved using the Fibonacci wavelet operational matrix method, which transforms the model into a system of algebraic equations for improved computational efficiency. The basic reproduction number R_0 is derived as a threshold parameter, indicating disease extinction for $R_0 < 1$ and persistence for $R_0 > 1$. Numerical simulations reveal that the fractional order significantly influences disease dynamics, with smaller orders yielding slower evolution and larger orders accelerating transmission. The results demonstrate the effectiveness of the proposed method and its potential applicability to more complex epidemic models incorporating additional control and environmental factors. Future research may focus on extending the model by incorporating vaccination strategies, stochastic effects, environmental variability, and optimal control measures to enhance its applicability in real-world epidemic management.

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