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QUATERNION FORMULATION OF CLASSICAL ELECTRODYNAMICS IN THE ABSENCE OF MAGNETIC MONOPOLE

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Keywords	*Abstract*
Quaternion, Classical Electrodynamics, Magnetic Monopole, Four Potential.	This study presents a quaternion formulation of classical electrodynamics in the absence of magnetic monopoles. The primary objective is to reformulate Maxwell's theory using quaternion algebra while preserving the physical predictions of classical electrodynamics. The analysis focuses on expressing electromagnetic quantities, including the four-current, four-potential, differential operator, and field equations, within a unified quaternion framework.The principal contribution of this work is the reformulation of Maxwell's equations in quaternion form. Rather than employing multiple independent vector equations, the quaternion formalism consolidates them into a compact set of equations without altering the physical content of Maxwell's theory. This approach simplifies the mathematical structure while maintaining equivalence with the classical formulation. The electric field, magnetic field, and propagation vector remain mutually perpendicular, demonstrating that the quaternion approach is consistent with classical

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	electromagnetic wave theory. Additionally, the quaternion equations reduce the number of separate mathematical expressions while preserving Lorentz covariance. This compact formulation is both mathematically elegant and well-suited for relativistic electrodynamics.

1. Introduction:

The equations of Maxwell [1] have been known in electrodynamics since 1865. Later, Oliver Heaviside [2] and William Gibbs [3] transformed these equations into today's most commonly used vector notation. Mathematically, quaternion represent a natural extension of complex numbers, forming an algebra under addition and multiplication [4]. Sir William Rowan Hamilton invented quaternion in 1843 and immediately sought to apply them to physics [5]. Since a quaternion has four components, all the components of a 4-vector can be included in it. However, one component (e.g., time) must be a purely imaginary number. Such a quaternion is called a Minkowski quaternion [6]. The difficulty with Minkowski quaternion is that they do not form an algebraic ring. The formulation of physical law using real quaternion has then be replaced by complex ones, and it has been recognized that complex quaternion represent a powerful instrument in formulating classical physical laws [7].

Keeping these facts in mind, we describe classical and quaternion electrodynamics in terms of differential equations in the absence of magnetic charge (i.e., a monopole). We describe the quaternion forms of four-current, four-potential, and quaternion differential operator and derive the generalized field equations, Maxwell's equations in the absence of a monopole, in a simple, compact, and consistent manner. Accordingly, other quantum equations, i.e., the field equation, the current equation, and the field tensor, are derived consistently.

In electrodynamics, we discuss the transverse nature of the electromagnetic waves. In quaternion form, the field equations reduce to a single equation, and this representation shows that the four-vector is similar to its transformation to a superluminal frame via complex superluminal Lorentz transformations. In this case, complex quantum mechanics for time-like events is visualized as quaternion quantum mechanics for space-like events, or vice versa. It is considered that, in quaternion form, the field equations may directly be extended from the theory of bradyons to the theory of tachyons or vice versa.

2. Objectives

The paper introduces a quaternion-based reformulation of classical electrodynamics, starting from Maxwell's equations and developing compact quaternion representations of electromagnetic quantities. The paper aims to represent the four-current, four-potential differential operators, and field equations in quaternion form while preserving Maxwell's theory in the absence of magnetic monopoles.

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Main Objectives of the Paper are

1. To formulate generalized potential field equations with scalar and vector potential form from Maxwell's equations.
2. To derive the Covariant form of Maxwell's equations with the electromagnetic field tensor and four-current density.
3. To formulate electric and magnetic field wave equations from Maxwell's equations and analyze the transverse nature of electromagnetic waves
4. To obtain Maxwell's equations in a single compact quaternion equation with four-current and four-potential of electromagnetic fields.

3. Classical Electrodynamics in absence of magnetic monopole

Considering Maxwell's equations governing classical electrodynamics in the absence of magnetic monopoles [8],

$$\vec{\nabla} \cdot \vec{E} = \rho_e \tag{1a}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \tag{1b}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} \tag{1c}$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \tag{1d}$$

Where is the electric field, is the magnetic field, is the electric charge density, and (magnetic charge density remains zero throughout this paper). is the electric current source density. Introducing the four-potential derived from the source term and,

Maxwell's equation (1.b) is Gauss's law for magnetism. Any vector field whose divergence is zero can always be written as the curl of another vector. i.e.

Taking the curl of equation (1.c)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} = -\frac{\partial (\nabla \times \vec{A})}{\partial t} = \nabla \times \frac{\partial \vec{A}}{\partial t}$$

A vector field with zero curl is the gradient of a scalar function.

$$E + \frac{\partial \vec{A}}{\partial t} = -\nabla \phi_e$$

$$E = -\nabla \phi_e - \frac{\partial \vec{A}}{\partial t}$$

From Maxwell's equations, electric and magnetic fields can be written in the following manner,

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$$\vec{E} = -\vec{\nabla}\phi_e - \frac{\partial \vec{A}}{\partial t} \quad (2.a)$$

$$\vec{H} = \vec{\nabla} \times \vec{A} \quad (2.b)$$

Thus, the generalized potential field equations with scalar and vector potentials follow from the generalized Maxwell equations (1.b and 1.c).

The vector wave function associated with the electromagnetic field may now be described as,

$$\vec{\psi} = \vec{E} - i\,\vec{H} \quad . \quad (3)$$

The Equations (2) and (3) lead to the following relation between the vector wave function and the components of the four-potential, i.e.

$$\vec{\psi} = \vec{E} - i\,\vec{H} = -\frac{\partial \vec{A}}{\partial t} = -\nabla\phi - \vec{\nabla} \times \vec{A}$$

$$\vec{\psi} = -\vec{\nabla}\phi_e - \frac{\partial \vec{A}}{\partial t} - i\vec{\nabla} \times \vec{A} \quad . \quad (4)$$

The divergence and curl of equation (3), and using equation (2)**,** we get the following equations,

$$\vec{\nabla} \cdot \vec{\psi} = \vec{\nabla}.(\vec{E} - i\vec{H}) = \vec{\nabla}.\vec{E} - i\left(\vec{\nabla}.\vec{H}\right)$$

$$\vec{\nabla} \cdot \vec{\psi} = \rho_e \quad (5.a)$$

$$\vec{\nabla} \times \vec{\psi} = \vec{\nabla} \times (\vec{E} - i\vec{H}) = \vec{\nabla} \times \vec{E} - i(\nabla \times \vec{H}) \quad .$$

$$\vec{\nabla} \times \vec{\psi} = -\frac{\partial \vec{H}}{\partial t} - i(j + \frac{\partial \vec{E}}{\partial t}) = i\,j - i(\frac{\partial \vec{E}}{\partial t} - i\frac{\partial H}{\partial t})$$

$$\vec{\nabla} \times \vec{\psi} = i\,\vec{j} - i\frac{\partial \vec{\psi}}{\partial t} \quad (5.b)$$

In covariant notation, the electric and magnetic fields of equations (2.a and 2.b) are the components of the following electromagnetic field tensor; the tensor is anti symmetric, Lorentz covariant, and contains both fields.

$$F_{\mu\upsilon} = \partial_\upsilon A_\mu - \partial_\mu A_\upsilon, \left(\mu, \upsilon = 0,1,2,3\right)$$

$$F_{\mu\upsilon} = -F_{\nu\mu} \quad (6)$$

where

$$F_{0i} = E^i$$

$$F_{ij} = \varepsilon_{ijk} H^k \quad . \quad (7)$$

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Using the electromagnetic field tensor, Maxwell's equations can be written in compact covariant form as follows.

$$F_{\mu\upsilon,\upsilon} = \partial^{\upsilon} F_{\mu\upsilon} = j_{\mu} \tag{8a}$$

$$F^{d}_{\mu\upsilon,\upsilon} = \partial^{\upsilon} F^{d}_{\mu\upsilon} = 0 \tag{8b}$$

where $\{j_{\mu}\} = \{\rho_e, \vec{j}\}$ is the four-current density and the electromagnetic field tensor $F_{\mu\upsilon}$ dual electromagnetic field tensor $F^{d}{}_{\mu\upsilon}$ are defined as,

$$F_{\mu\nu} = \begin{vmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & H_z & -H_y \\ -E_y & -H_z & 0 & H_x \\ -E_z & H_y & -H_x & 0 \end{vmatrix}$$

$$F^{d}{}_{\mu\upsilon} = \frac{1}{2}\varepsilon_{\mu\upsilon\sigma\lambda} F^{\sigma\lambda} \quad . \tag{9}$$

Here d represent the dual part, while $\varepsilon_{\mu\upsilon\sigma\lambda}$ is completely anti-symmetric tensor of rank 4.

The electric and magnetic field given of equations (2.a and 2.b) are invariant under gauge transformation

$$A' = A + \nabla\Lambda$$

$$\phi_e' = \phi_e - \frac{\partial\Lambda}{\partial t}$$

The Lorentz gauge simplifies Maxwell's equations to wave equations that preserve Lorentz covariance.

The Wave equation for the scalar potential by using equations (1.a and 2.a) we get

$$\vec{\nabla}\cdot\vec{E} = \rho_e$$

$$\vec{\nabla}\cdot(-\vec{\nabla}\varphi_e - \frac{\partial\vec{A}}{\partial t}) = \rho_e$$

$$\nabla^2\varphi_e + \frac{\partial(\vec{\nabla}.\vec{A})}{\partial t} = -\rho_e$$

After imposing Lorentz condition $\vec{\nabla}\cdot\vec{A} + \frac{\partial\vec{\phi}_e}{\partial t} = 0$

$$\nabla^2\varphi_e + \frac{\partial}{\partial t}(-\frac{\partial\phi_e}{\partial t}) = -\rho_e$$

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$$\nabla^2 \varphi_e - \frac{\partial^2 \phi_e}{\partial t^2} = -\rho_e$$

$$\Box \phi_e = -\rho_e \tag{10}$$

Here $\Box$ represents the D'Alembertian operator i.e.

$$\Box = \nabla^2 - \frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2}$$

The Wave equation of vector potential, by using equations (1.d, 2.a and 2.b), we get

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{j} + \frac{\partial}{\partial t}(-\vec{\nabla}\phi_e - \frac{\partial \vec{A}}{\partial t})$$

$$\vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A} = \vec{j} - \vec{\nabla}\frac{\partial \phi_e}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2}$$

After imposing Lorentz condition $\vec{\nabla} \cdot \vec{A} + \frac{\partial \vec{\phi}_e}{\partial t} = 0$

$$\nabla^2 \vec{A} - \frac{\partial^2 \vec{A}}{\partial t^2} = -\vec{j}$$

$$\Box \vec{A} = -\vec{j} \tag{11}$$

These wave equations (10 and 11) define the scalar and vector potentials, whose sources are charge and current densities. Electromagnetic disturbances therefore propagate. Combining the two wave equations (10 and 11) into a single Lorentz-covariant equation shows that charge and current act together as a single four-current that generates the four-potential.

$$\Box A_\mu = -\vec{j}_\mu \tag{12}$$

Where $\{\vec{A}_\mu\} = \{\phi_e, \vec{A}\}$ is four-potential and $\{\vec{j}_\mu\} = \{\rho_e, \vec{j}\}$is four-current.

Where is the four-potential and is the four-current.
It represents the complete distribution and transport of electric and magnetic charges, thereby extending the classical concept of electric current without altering Maxwell's theory in the limit where magnetic charges vanish. This unified source description forms the basis for the quaternion formulation of the generalized electromagnetic field equations developed in the subsequent sections.
Taking the divergence of equation (1.d) and using equation (1.a)

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$$\vec{\nabla}\times\vec{H} = \vec{j} + \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}.(\vec{\nabla}\times\vec{H}) = \vec{\nabla}.\vec{j} + \vec{\nabla}.\frac{\partial \vec{E}}{\partial t} = \vec{\nabla}.\vec{j} + \frac{\partial(\vec{\nabla}.\vec{E})}{\partial t}$$

$$\vec{\nabla}.\vec{j}_e + \frac{\partial \rho_e}{\partial t} = 0 \tag{13}$$

This is the continuity equation for electric charge. It is essential for the consistency of Maxwell's equations.

Taking the curl of equation (5.b), we may introduce new parameters (i.e., the field current) as,

$$\vec{\nabla}\times\vec{\psi} = i\,\vec{j}_e - i\frac{\partial \vec{\psi}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{\psi}) = \vec{\nabla}\times(i\,\vec{j}_e - i\frac{\partial \vec{\psi}}{\partial t}) = i\,(\vec{\nabla}\times\vec{j}_e) - i\,(\vec{\nabla}\times\frac{\partial \vec{\psi}}{\partial t})$$

$$\vec{\nabla}\,(\vec{\nabla}.\vec{\psi}) - \nabla^2\psi = -i(\vec{\nabla}\times\vec{j}_e) - i\frac{\partial(\vec{\nabla}\times\vec{\psi})}{\partial t}$$

Using equation (5.a) and (5.b)

$$\vec{\nabla}\rho_e - \nabla^2\psi = -i(\vec{\nabla}\times\vec{j}_e) - i\frac{\partial(\vec{\nabla}\times\vec{\psi})}{\partial t}$$

$$\vec{\nabla}\rho_e - \nabla^2\psi = -i(\vec{\nabla}\times\vec{j}_e) - \frac{\partial \vec{j}_e}{\partial t} - \frac{\partial^2\psi}{\partial t^2}$$

$$\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = -i(\vec{\nabla}\times\vec{j}_e) - \frac{\partial \vec{j}_e}{\partial t} - \vec{\nabla}\rho_e$$

$$\Box\vec{\psi} = \vec{S} \tag{14}$$

Where $\vec{S} = -\frac{\partial \vec{j}_e}{\partial t} - i\left(\vec{\nabla}\times\vec{j}_e\right) - \vec{\nabla}\rho_e$

This is the inhomogeneous wave equation for the electromagnetic field with the combined source called the field current. Instead of solving four equations (1.a-1.d) separately, this paper combines them into a single compact wave equation, which is mathematically simpler and especially useful for generalizing classical electrodynamics.

The field current represents how electromagnetic fields propagate and interact in space. It is mainly a mathematical construct that makes the quaternion equation more compact. It provides the bridge between ordinary vector electrodynamics and quaternion electrodynamics.

The plane wave solutions that satisfy Maxwell's equations are used to demonstrate the properties of electromagnetic waves. The wave forms of electric and magnetic fields are

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$$\nabla^2 \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{15}$$

$$\nabla^2 \vec{H} - \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \tag{16}$$

Plane wave solutions of Maxwell's field equations given by equations (15 and 16) may be written as:

$$\vec{E} = \vec{E}(0)\exp i\left(\vec{k}\cdot\vec{x} - \omega t\right) \tag{17}$$

$$\vec{H} = \vec{H}(0)\exp i\left(\vec{k}\cdot\vec{x} - \omega t\right). \tag{18}$$

where $\vec{k}$ is the propagation vector $\left|\vec{k}\right| = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$ is denotes the angular velocity, λ is the wave length of electromagnetic waves.

For a plane wave, the differential operator becomes

$$\nabla \rightarrow ik \tag{17.a}$$

$$\frac{\partial}{\partial t} \rightarrow -i\varpi \tag{17.b}$$

Now Maxwell's equations (1.a-1.d) become

$$ik.\vec{E} = 0 \tag{18.a}$$

$$ik.\vec{B} = 0 \tag{18.b}$$

$$ik \times \vec{E} = i\omega\vec{H} \tag{18.c}$$

$$ik \times \vec{H} = -i\omega\vec{E} \tag{18.d}$$

Substituting solutions (17 and 18) into the divergence equations (18.a-18.b) we obtain,

$\vec{k}\cdot\vec{E} = 0$ and $\vec{H}\cdot\vec{E} = 0$

The electric and magnetic fields are not along the direction of propagation, which implies that-

-

$$\vec{k} \perp \vec{E} \quad \text{and} \quad \vec{H} \perp \vec{E}. \tag{19}$$

Considering curl equations (18.c-18.d), we obtain

$k \times \vec{E} = \omega\vec{H}$ and $k \times \vec{H} = -\omega\vec{E}$

The electric and magnetic fields are perpendicular to the wave propagation, which implies that

$$\vec{E} \perp \vec{k} \quad \text{and} \quad \vec{H} \perp \vec{k}. \tag{20}$$

where $\perp$ denotes the perpendicular sign. The three vectors electric, magnetic, and wave are mutually orthogonal to each other. Hence, electromagnetic waves are transverse in nature

4. Quaternion Classical Electrodynamics

Sir R.W. Hamilton [9-11] discovered the generalization from the basic elements (1, i) of a complex number field to the basis (1, i, j, k) of a quaternion number field. Hamilton's generalization then led to a great discovery in the history of mathematics, though it has not been taken seriously until the present time.

A quaternion is thus defined as

$$q = q_0 + q_1 e_1 + q_2 e_2 + q_3 e_3 \tag{21}$$

where q_0, q_1, q_2 *and* q_3, are real numbers. The quaternion units e_1, e_2, e_3 satisfy the following multiplication rule,

$$e_0^2 = 1$$

$$e_j e_k = -\delta_{jk} e_0 + \varepsilon_{jkl} e_l \tag{22}$$

where ε_{jkl} is Levi-Civita three index symbols and δ_{jk} is Kronecker delta. A quaternion is essentially an ordered tetrad number, and then we may write equation (21) as,

$$q = (q_0, q_1, q_2, q_3) \tag{23}$$

with the understanding that it is immaterial whether we write $q_i e_i$ or $e_i q_i$ and so on. Quaternion conjugate is defined as,

$$\overline{\overline{q}} = q_0 - e_1 q_1 - e_2 q_2 - e_3 q_3 \tag{24}$$

Then from equation (20), we get

$$(e_1, e_1) = (e_2, e_2) = (e_3, e_3) = 1$$

$$(e_2, e_3) = (e_3, e_2) = (e_3, e_1) = (e_1, e_3) \tag{25}$$

$$= (e_1, e_2) = (e_2, e_1) = 0$$

which shows that e_0, e_1, e_2, e_3 may be regarded as orthogonal triad of unit vectors in Euclidian 3-space ,

Before introducing the quaternion four-current and four-potential, it is useful to note why quaternion is employed in the present formulation. Quaternion algebra provides a natural framework in which these scalar and vector components can be combined into a single mathematical object. This unified representation reduces the number of separate equations and allows many electromagnetic relations to be written in a more compact and elegant form. Consequently, the quaternion formulation offers a concise and unified

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description of electromagnetic fields and their sources, motivating the introduction of the quaternion four-current and four-potential in the following equations.

We may write the four-current $\{j_\mu\}$ and four-potential $\{A_\mu\}$ of electromagnetic fields in terms of quaternion as [12],

$$j = -i\rho_e + e_1 j_1 + e_2 j_2 + e_3 j_3 \tag{26}$$

$$A = -i\phi_e + e_1 A_1 + e_2 A_2 + e_3 A_3 \tag{27}$$

Equations (26) and (27) introduce the quaternion representation of the electromagnetic four-current and four-potential. These equations are the foundation of the quaternion formulation because they combine scalar and vector quantities into a single mathematical object

Accordingly, the four differential operators may be written as quaternions in the following manner, i.e.

$$\partial = -i\partial_t + e_1\partial_1 + e_2\partial_2 + e_3\partial_3$$

$$\text{where} \quad \partial_t = \frac{\partial}{\partial t}, \partial_1 = \frac{\partial}{\partial x} = \partial_x, \partial_2 = \frac{\partial}{\partial y} = \partial_y, \partial_3 = \frac{\partial}{\partial z} = \partial_z \tag{28}$$

The classical four-current, four-potential, and four-dimensional differential operators are expressed as quaternion. Applying the quaternion differential operator to these quantities yields compact quaternion field equations, along with the Lorentz gauge and continuity equations.

Now, operating equation (28) on equations (26) and (27) and using the multiplication rules (22), we get;

$$\begin{aligned} \partial \mathrm{A} = & -(\partial_t\phi_e + \partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3) - ie_1\{-\partial_t A_1 + \partial_1\phi_e - i(\partial_2 A_3 - \partial_3 A_2)\} \\ & - ie_2\{-\partial_t A_2 + \partial_2\phi_e - i(\partial_3 A_1 - \partial_1 A_3)\} - ie_3\{-\partial_t A_3 + \partial_3\phi_e - i(\partial_1 A_2 - \partial_2 A_1)\} \end{aligned} \tag{29}$$

and

$$\begin{aligned} \partial \mathrm{j} = & -(\partial_t\rho_e + \partial_1 j_1 + \partial_2 j_2 + \partial_3 j_3) - ie_1\{-\partial_t j_1 + \partial_1\rho_e - i(\partial_2 j_3 - \partial_3 j_2)\} \\ & - ie_2\{-\partial_t j_2 + \partial_2\rho_e - i(\partial_3 j_1 - \partial_1 j_3)\} - ie_3\{-\partial_t j_3 + \partial_3\rho_e - i(\partial_1 j_2 - \partial_2 j_1)\} \end{aligned} \quad . \tag{30}$$

Equations (29) and (30) are reduced to the following compact quaternion equations as,

$$\partial \mathrm{A} = -\psi_t - ie_1\psi_1 - ie_2\psi_2 - ie_3\psi_3 \tag{31}$$

$$\partial \mathrm{j} = -S_t - ie_1 S_1 - ie_2 S_2 - ie_3 S_3 \tag{32}$$

where

$$\psi_{\mathrm{t}} = \partial_t\phi_e + \partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 = 0 \tag{33}$$

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(Lorentz condition)

and $S_t = \partial_t \rho_e + \partial_1 j_1 + \partial_2 j_2 + \partial_3 j_3 = 0$. (34)

(Continuity condition)

Now using the following quaternion forms of ψ and S as

$$\psi = -\psi_t - i(e_1\psi_1 + e_2\psi_2 + e_3\psi_3) \quad (35)$$

$$S = -S_t - i(e_1 S_1 + e_2 S_2 + e_3 S_3) \quad (36)$$

we get

$$\partial \mathrm{A} = \psi \quad (37)$$

$$\partial \mathrm{j} = S . \quad (38)$$

These are the quaternion forms of field equations (4) and (14) for electromagnetic potential and current, respectively. These equations are important because they transform lengthy vector expressions into compact quaternion equations while preserving the complete physical content of Maxwell's equations.

The conjugate of quaternion field equations (35) and (36) may also be written as;

$$\bar{\partial}\bar{A} = \bar{\psi} \quad (39)$$

$$\bar{\partial}\bar{j} = \bar{S} \quad (40)$$

where $\bar{\partial}, \bar{A}, \bar{\psi}, \bar{j}, \bar{S}$ denotes as the quaternion conjugate i.e.

$$\bar{\partial} = -i\partial_t - (e_1\partial_1 + e_2\partial_2 + e_3\partial_3) \quad (41)$$

$$\bar{A} = -i\phi_e - (e_1 A_1 + e_2 A_2 + e_3 A_3) \quad (42)$$

$$\bar{j} = -i\rho_e - (e_1 j_1 + e_2 j_2 + e_3 j_3) \quad (43)$$

$$\bar{\psi} = -\psi_t + i(e_1\psi_1 + e_2\psi_2 + e_3\psi_3) \quad (44)$$

$$\bar{S} = -S_t + i(e_1 S_1 + e_2 S_2 + e_3 S_3) \quad . \quad (45)$$

Using equations (28, 41), (27, 42) and (26, 43) and the similar equation for quaternion momentum p, we get

$$\bar{\partial}\partial = \partial\bar{\partial} = -\partial_t^2 + \partial_1^2 + \partial_2^2 + \partial_3^2 = -\Box \quad (46)$$

$$\bar{A}A = A\bar{A} = |A|^2 = -\phi_e{}^2 + A_1^2 + A_2^2 + A_3^2 = -\phi_e{}^2 + |\vec{A}|^2 \quad (47)$$

$$\bar{j}j = j\bar{j} = |j|^2 = -\rho_e{}^2 + j_1^2 + j_2^2 + j_3^2 = -\rho_e{}^2 + |\vec{j}|^2 \quad (48)$$

$$\bar{p}p = p\bar{p} = |p|^2 = -E^2 + p_1^2 + p_2^2 + p_3^2 = -E^2 + |\vec{p}|^2 \quad (49)$$

which shows that any time-like interval in usual space becomes space-like in quaternion form.

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The quaternion forms of Maxwell's equations given by equations (37) and (38) remain invariant under quaternion transformations corresponding to Lorentz transformations;

$$q' = -\bar{u}\,\bar{q}\,u^{*} \quad \text{with} \quad u\bar{u} = 1. \tag{50}$$

Instead of four separate Maxwell's wave equations (12), the quaternion equation combines them into a single compact equation that is exactly the standard wave equation for the electromagnetic potential in the Lorentz gauge.

$$\bar{\partial}\partial A = -j \tag{51}$$

The similar result may be obtained for equation (14) in the following quaternionic form;

$$\bar{\partial}\partial\psi = -S \tag{52}$$

5. Discussion and Conclusion

This work examines electrodynamics in the absence of magnetic charge. Maxwell's equations are derived without monopoles. Additional quantum equations, including the generalized field equations, the electromagnetic field tensor, the continuity equation, and the field-current equations, are also addressed within the context of electrodynamics. The plane-wave solution for electric and magnetic fields illustrates the transverse nature of the electromagnetic field. The quaternion representation of the field equations for the generalized potential and current is analyzed. The norm of the quaternion differential operator, four-potential, four-current, and four-momentum indicates that any time-like interval in conventional space becomes space-like in quaternion form. The quaternion forms of Maxwell's equations for electrodynamics remain invariant under quaternion Lorentz transformations. It is further concluded that complex quantum mechanics for time-like events corresponds to quaternion quantum mechanics for space-like events. The quaternion formalism offers a promising mathematical framework for extending classical electrodynamics to more advanced theories. Since quaternion naturally combine scalar and vector quantities, they may simplify the formulation of relativistic field equations and support future research in quaternion quantum mechanics, covariant field theory, and other generalized electromagnetic models.

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