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Optimal production policy for imperfect items under learning and forgetting environment

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Keyword

Imperfect, Exponential demand, Partial Backlogging, Inflation, Learning, Forgetting

Abstract

In an imperfect production process with the impact of inflation, this study studies an economic production quantity (EPQ) model with exponential demand. The machine may go from an in-control state to an out-of-control state throughout a lengthy procedure. As a result, the system produces imperfect items. Shortages are permitted and partially backlogged with an exponential rate. We developed the model in both fuzzy and crisp sense. It is well known that as the number of cycle increase the percentage of defective items decrease due to the learning and forgetting effect of the past. Similar in the case of set up cost which also decreases due to the learning and forgetting effect.

INTRODUCTION

The basic goal of inventory control issues is to maximise order quantity or lot size while taking capacity constraints into account. In such situations, the goal is to either maximise the system's positive effects or reduce the entire expenses of the inventory control system, including setup, ordering, and holding charges.

The fundamental economic order quantity model was first modified by Porteus (1986) to account for the impact of damaged goods. Rosenblatt and Lee first considered defective goods as a result of examining a production process that was not flawless in terms of quality. They extended the work done for imperfect quality items under random yield and developed economic order quantity which contradicts with the findings of Rosenblatt and Lee (1986) that the economic lot size quantity tends to decrease as the average percentage of imperfect quality items increases. Mandal and Phaujdar (1989) and others have considered the deterioration of items along with the stock-dependent demand or time-varying demand. Mandal and Maiti (1997) has considered an inventory model for the items made of these materials with stock-dependent demand and shortages. When the demand rate is a linearly declining function of the selling price, Wee and Law (1999) developed a deteriorating inventory model for estimating the size of an economic production lot under inflationary conditions. Salameh and Jaber (2000) presented a model for items with imperfect quality. Goyal and Cardenas-Barron (2002) presented a simple approach for determining the economic production quantity for Salameh and Jaber (2000). For degrading goods, Balkhi (2004) presented two flexible production lot-size inventory models in which the production rate at any given moment relies on the demand and the quantity of on-hand inventory at that precise moment. Maiti and Maiti (2005) developed a production policy for damageable items with variable cost function in an imperfect production process via genetic algorithm. Papachristos and Konstantaras (2006) looked at the issue of non-shortages in model with proportional imperfect quality, when the proportion of the imperfects is a random variable. Singh, Kumar and Gaur (2007) investigated an EPQ model for deteriorating items with variable demand, partial backlogging and inflation. Singh and Singh (2008) developed a production model for items under the effect of inflation and permissible delay in payments.

The "learning curve" introduced by Wright (1936), was the first attempt to link the performance in a specific task to the number of times that task is repeated. Jabber and Bonney (2003) developed a lot sizing model with learning forgetting in set-ups. Jawla and Singh (2016) developed an inventory model with imperfect production, for deteriorating items under learning and inflationary environment. Sepehri (2021) presented a production-inventory paradigm with imperfect quality that is supported by investments in quality enhancement and preservation technologies.

Assumptions and Notations:

The following assumptions are made herein:

The inventory system under consideration deals with single item.

1) The demand rate $D(t) = ae^{bt}$ is deterministic and is a known function of time; the function $D(t)$.
2) A fraction $\theta(t) = \theta\ t$ of the on hand inventory deteriorates per unit time and deterioration rate item.
3) The inventory system is an imperfect production system and production rate is given by $(t) = p(1-e)$, where p is production rate per unit time and $0 \le e < 1, P(t) > D(t)$.
4) The unit production cost is a function of the rate of production and given by $C_P = R + \frac{G}{P} + HP$.
5) Shortages are permitted and partially backlogged. Effect of inflation is also considered.
6) The inventory setup cost follows the learning and forgetting effect, function of this cost When learning $C_O + \frac{C_{OS}}{n^\gamma}$, where C_O cost of the first set-up
   When learning and forgetting $C_O(y_j + 1)^{-\lambda}$, y_j is strength of memory at beginning of set-up j and λ is the learning coefficient.
7) Holding cost is taken to be a linear function of time i.e. variable in nature.
8) The planning horizon of the system is finite. Replenishment rate is infinite and lead-time is zero.

The following notations will be used throughout this study:

$D(t)$	: Demand rate
$\theta(t)$	: Deterioration rate of the item
$P(t)$	: Production rate
R	: Material cost per unit
G	: Constant factor associated with costs like labour, energy costs
H	: Constant factor associated with tool/die cost
$C_h + \alpha t$	: Holding cost per unit per unit time, where $\alpha > 0$.
C_P	: Production cost per unit per unit time
STC	: Setup cost per unit per unit time
C_L	: Lost sale cost per unit per unit time
C_I	: Inspection cost per unit per unit time
C_S	: Shortage cost per unit per unit time
C_R	: Rework cost per unit per unit time
C_d	: Deterioration cost per unit per unit time
S	: Selling price for the inventory system
n	: Cumulative number of shipments
r	: Constant representing the difference between the discount rate and inflation rate
B	: Backlogging rate, $B = e^{-\delta t}$, where δ is a positive constant.
t_0	: The time at which production cycle starts
t_1	: The time when production stops

t_2 : The time when inventory level reaches zero

t_3 : The time at which production restarts

T : The time for production cycle

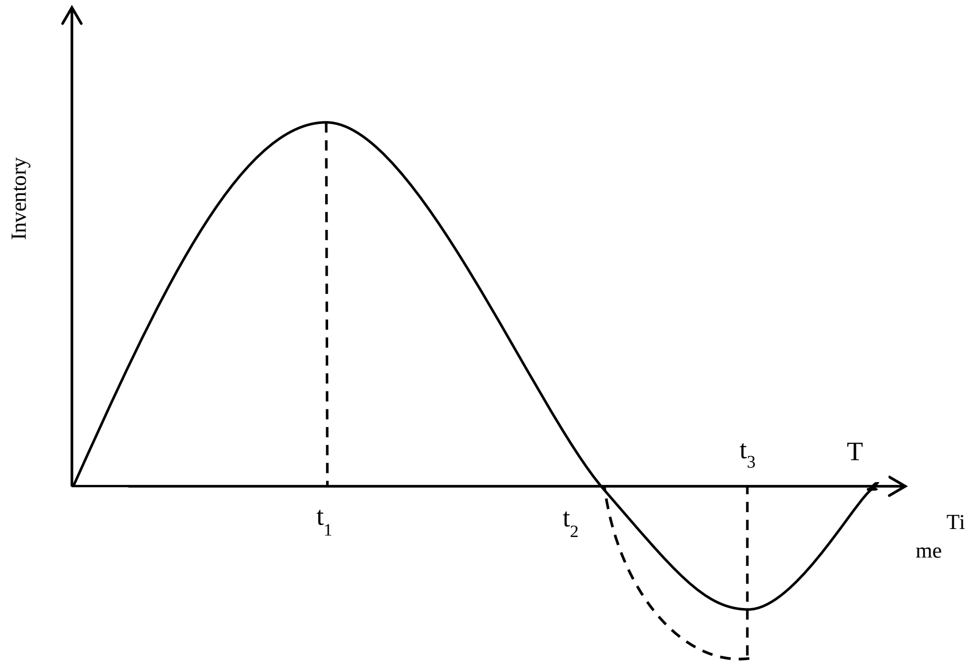


Fig 1: Inventory levels during cycle times

Model Formulation: Time dependent demand rate and deterioration rate

In this model demand rate, deterioration rate, backlogging rate and holding cost are function of time. Production rate is taken as constant. Then the inventory levels at any time during the time periods $[t_0, t_1]$, $[t_1, t_2]$, $[t_2, t_3]$, and $[t_3, T]$ are given by these differential equations:

$$I_1'(t) = p(1-e) - ae^{bt} - \theta(t)I_1(t) \qquad t_0 \le t \le t_1 \qquad (1)$$

$$I_2'(t) = -ae^{bt} - \theta(t)I_2(t) \qquad t_1 \le t \le t_2 \qquad (2)$$

$$I_3'(t) = -e^{-\delta t}ae^{bt} \qquad t_2 \le t \le t_3 \qquad (3)$$

$$I_4'(t) = p(1-e) - ae^{bt} \qquad t_3 \le t \le T \qquad (4)$$

With the initial and boundary conditions

$$I_1(0) = 0,\ I_2(t_2) = 0,\ I_3(t_2) = 0 \ and\ I_4(T) = 0 \qquad (5)$$

These first order differential Equations can be solved by using above boundary conditions and solutions are:

$$I_1(t)=\left[p(1-e)\left(t+\frac{\theta}{6}t^3\right)-a\,(t+\frac{b}{2}t^2+\frac{\theta}{6}t^3)\right]e^{-\theta t^2/2}, \qquad t_0 \le t \le t_1 \qquad (6)$$

$$I_2(t)=\left[a\left\{(t_2-t)+\frac{b}{2}\left(t_2{}^2-t^2\right)+\frac{\theta}{6}\left(t_2{}^3-t^3\right)\right\}\right]e^{-\theta t^2/2} \qquad t_1 \le t \le t_2 \qquad (7)$$

$$I_3(t)=a\left[(t_2-t)+\left(\frac{b-\delta}{2}\right)\left(t_2{}^2-t^2\right)\right], \qquad t_2 \le t \le t_3 \qquad (8)$$

$$I_4(t)=\left[p(1-e)(t-t_4)-\frac{a}{b}(e^{bt}-e^{bt_4})\right], \qquad t_3 \le t \le T \qquad (9)$$

At $t=t_1$, $I_1(t_1)=I_2(t_1)$ then from equations (6) and (7), we get

$$t_2=f(t_1)$$

At $t=t_3$, $I_3(t_3)=I_4(t_3)$ then from equations (8) and (9), we get

$$T=f(t_1,t_3)$$

In the present inventory system, the present value of sales revenue during the interval $[t_0,T]$ is

$$S_a=S\left[\int_{t_0}^{t_1}ae^{bt}e^{-rt}dt+\int_{t_1}^{t_2}ae^{bt}e^{-rt}dt+\int_{t_2}^{t_3}ae^{bt}e^{-\delta t}e^{-rt}dt+\int_{t_3}^{T}ae^{bt}e^{-rt}dt\right] \qquad (10)$$

The production occurs during $[t_0,t_1]$ and $[t_3,T]$. The present worth of the production cost is

$$PC=(R+\frac{G}{P}+HP)\left[\int_{t_0}^{t_1}Pe^{-rt}dt+\int_{t_3}^{T}Pe^{-rt}dt\right] \qquad (11)$$

The inventory holding cost in the interval $[t_0,t_2]$ is

$$HC=\left[\int_{t_0}^{t_1}(C_h+\alpha t)I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}(C_h+\alpha t)I_2(t)e^{-rt}dt\right] \qquad (12)$$

The inspection cost of the given inventory system is

$$IC=C_I\left[\int_{t_0}^{t_1}p(1-e)e^{-rt}dt+\int_{t_3}^{T}p(1-e)e^{-rt}dt\right] \qquad (13)$$

Shortages occur during the interval $[t_2,T]$. Then the present worth of the shortage cost is

$$SC=\left[\int_{t_2}^{t_3}(-I_3(t))e^{-rt}dt+\int_{t_3}^{T}(-I_4(t))e^{-rt}dt\right] \qquad (14)$$

Deterioration of items occurs during the interval $[t_0,t_2]$. The cost for deteriorated unit is

$$DC = \left[\int_{t_0}^{t_1} p(1-e)e^{-rt} dt - \int_{t_0}^{t_2} ae^{bt} e^{-rt} dt \right] \quad (15)$$

Lost sales occurs during the interval $[t_2, t_3]$. Then the opportunity cost due to lost sales is

$$LC = C_L \left[\int_{t_2}^{t_3} (1-e^{-\delta t}) ae^{bt} e^{-rt} dt \right] \quad (16)$$

The present worth of the rework cost is given by

$$RC = C_R \left[\int_{t_0}^{t_1} p(1-e)e^{-rt} dt + \int_{t_3}^{T} p(1-e)e^{-rt} dt \right] \quad (17)$$

Set up cost of the inventory system = STC (18)

Consequently, the total present value of the profit of the system during the replenishment cycle can be formulated by using the equations (10)-(18) as:

$$NP = \frac{1}{T}[S_a - STC - PC - HC - IC - C_S.SC - C_d.DC - LC - RC] \quad (19)$$

Numerical Example and Sensitivity Analysis :

Suppose that there is a product with an exponentially increasing function of demand $D(t) = ae^{bt}$, where a and b are arbitrary constants satisfying $a > 0$ and $b > 0$. The remaining parameters of the inventory system are:

$$C_O = 500, C_{Os} = 0.05, C_h = 0.5, C_d = 2.5, C_s = 2, C_L = 5, C_I = 0.3, C_R = 0.4, S = 2000,$$
$$a = 2, b = 0.044, R = 1.5, G = 2.5, H = 0.01, P = 30.5, n = 1, r = 0.03, \theta = 0.02, \alpha = 0.02,$$
$$\delta = 0.01$$

When Full Transmission of Learning in Setup Cost, the optimal values of time t_1, t_2, t_3, T and maximum profit are:

$$t_1^* = 1.989,\ t_2^* = 30.332,\ t_3^* = 58.497,\ T^* = 60.473 \text{ and } NP^* = 2145.99$$

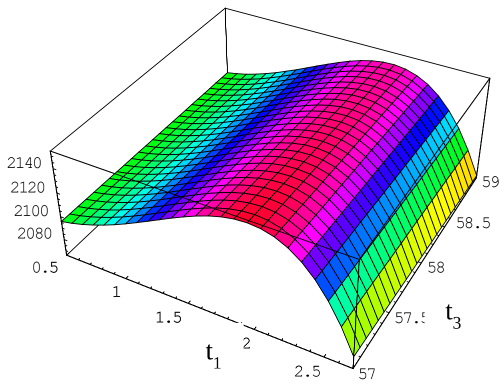
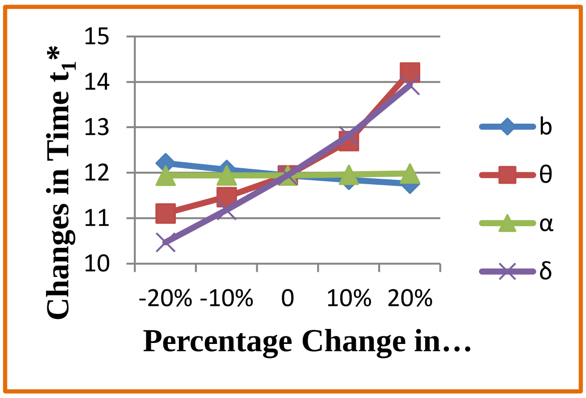
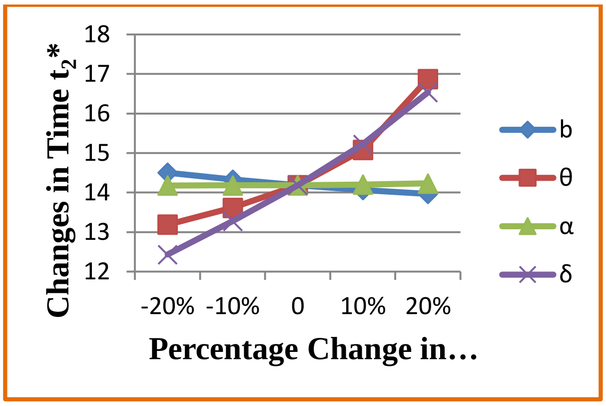


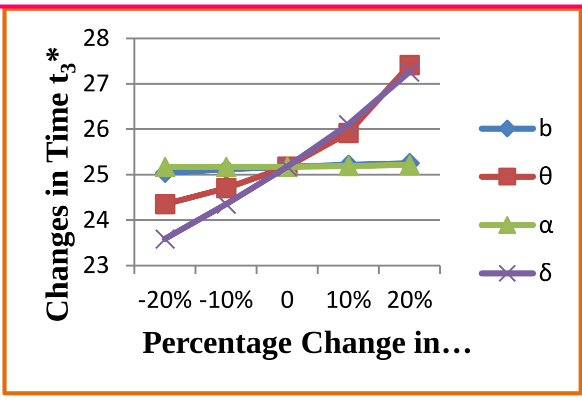
Fig.2. Concavity of the Net profit w.r.t. $t_1^*$and t_3^*

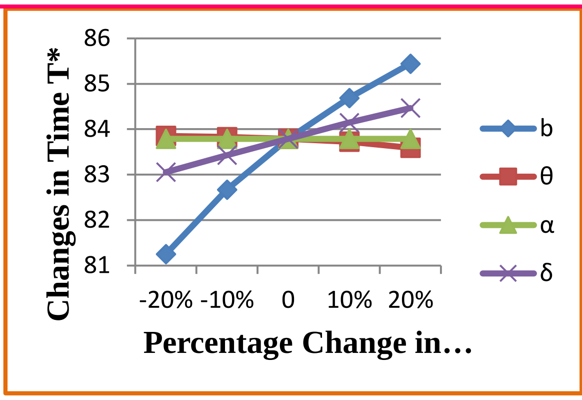
Sensitivity Analysis:

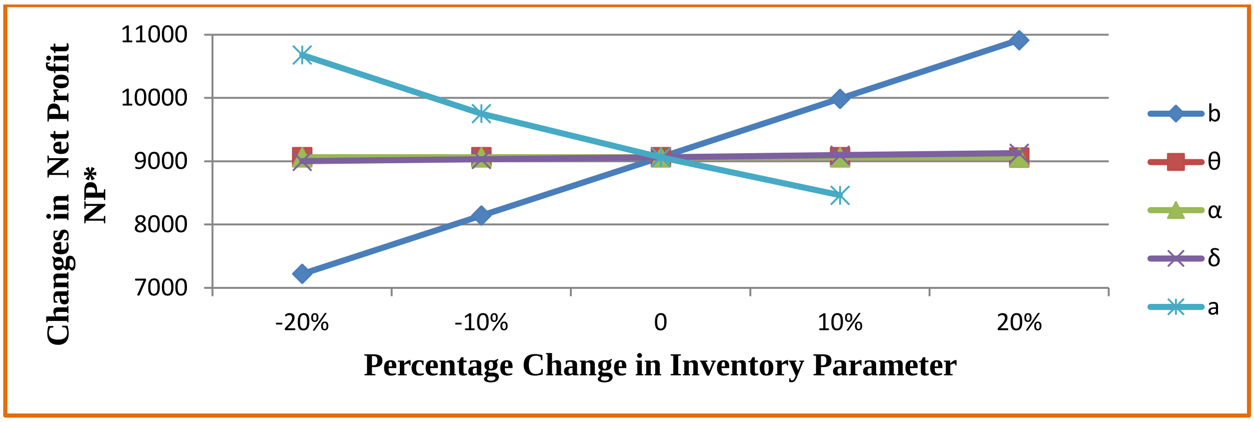
Parameters	% change	t_1^*	t_2^*	t_3^*	T^*	NP^*
	-20%	12.212	14.502	25.041	81.250	7220.73
	-10%	12.065	14.327	25.118	82.669	8140.59
b	0%	11.945	14.188	25.175	83.79	9063.23
	+10%	11.845	14.065	25.217	84.689	9987.83
	+20%	11.761	13.966	25.252	85.443	10913.9
	-20%	11.109	13.191	24.349	83.853	9064.27
	-10%	11.467	13.617	24.703	83.828	9063.80
$\boldsymbol{\theta}$	0%	11.945	14.188	25.175	83.790	9063.23
	+10%	12.699	15.080	25.919	83.726	9062.51
	+20%	14.212	16.876	27.411	83.593	9061.37
	-20%	11.939	14.178	25.169	83.79	9063.21
	-10%	11.942	14.181	25.172	83.79	9063.22
$\boldsymbol{\alpha}$	0%	11.945	14.188	25.175	83.79	9063.23
	+10%	11.961	14.203	25.190	83.79	9063.23
	+20%	11.985	14.232	25.215	83.79	9063.24
	-20%	10.469	12.431	23.583	83.055	8999.85
	-10%	11.178	13.273	24.351	83.429	9031.48
$\boldsymbol{\delta}$	0%	11.945	14.188	25.175	83.790	9063.23
	+10%	12.816	15.219	26.102	84.144	9095.06
	+20%	13.924	16.534	27.261	84.467	9126.93











Observations:

The present optimal value of net profit NP^* is more sensitive to consumption rate parameter b, as we increases the value of b, net profit NP^* will increase rapidly. Similarly, the increment in b leads to a negative change on the optimal value of ${t_1}^*$and ${t_2}^*$ and positive change on ${t_3}^*$ and T^*.

The increment in deterioration rate θ leads to a negative change on the present optimal value of T^* and net profit NP^*, i.e., net profit NP^* decreases with the increase of θ. Whereas positive change in deterioration rate θ results in a positive change in the optimum values of ${t_1}^*$, ${t_2}^*$and ${t_3}^*$.

When carrying cost parameter α will increase, the optimum values of ${t_1}^*$, ${t_2}^*$, ${t_3}^*$ and the present optimal value of net profit NP^*will increase. That is, the positive changes in α will cause the positive changes in ${t_1}^*$, ${t_2}^*$, ${t_3}^*$ and NP^*. Whereas T^* remains unchanged.

The positive change in backlog parameter δ leads to a positive change in the present optimal value of ${t_1}^*$, ${t_2}^*$, ${t_3}^*$, T^* and net profit NP^* i.e., when δ increases, ${t_1}^*$, ${t_2}^*$, ${t_3}^*$, T^* and net profit NP^* will increase.

Conclusion:

All manufacturing systems want to get more profit. To gain more profit, the machinery systems have to go through a long-run process. In long-run process due to machine breakdowns, labour problems, etc., the imperfect items are produced. This paper addressed a finite time-horizon production-inventory model for deteriorating items having exponential demand rate where both the perfect and imperfect items are produced. The perfect items are ready for sale and the imperfect quality items are reworked at a cost to make the original quality of the products and sale. The model permits inventory shortage in each cycle, which is partially backlogged within the cycle. Holding cost is taken as variable and setup cost follows the learning and forgetting effect. Optimal results of model are presented. The implementation of the proposed method was illustrated by using a numerical example. The sensitivity of the model was tested for a number of parameters. The results indicated that the annual profit tends to increase with the learning and forgetting effect on set up cost and annual profit is more sensitive to demand parameter *a*.

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